ALPHA CLASSES

Class 10 Mathematics

1. Real Numbers

NOTES

Topic	Reference Code
Real Numbers	C10-M-01-T06-NOTES-01

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Concept – The Big Picture

- Every composite number is made up of prime numbers multiplied together — primes are the "building blocks" (eent, neenv) of all natural numbers.
- The **Fundamental Theorem of Arithmetic** guarantees that this prime breakdown is unique (order ke alawa), so two people factorising the same number will always get the same set of primes.
- Once you know the prime factorisation of numbers, finding **HCF** and **LCM** becomes a simple, systematic process — no guesswork needed.
- HCF picks only the **common** primes with the **smallest** powers; LCM picks **all** primes with the **largest** powers.
- These ideas appear in almost every CBSE board paper — as direct questions, word problems, or as a step inside bigger proofs.

Key Definitions

Term	Definition
Prime Number	A natural number greater than 1 that has exactly two factors: 1 and itself. Examples: 2, 3, 5, 7, 11, 13 …
Composite Number	A natural number greater than 1 that has more than two factors. Examples: 4, 6, 8, 9, 12 …
Prime Factorisation	Writing a composite number as a product of prime numbers. Example: $60 = 2^2 \times 3 \times 5$
Fundamental Theorem of Arithmetic	Every composite number can be expressed (uniquely, apart from the order of factors) as a product of primes.
HCF (Highest Common Factor)	The greatest number that divides each of the given numbers exactly. Also called GCD.
LCM (Lowest Common Multiple)	The smallest positive number that is exactly divisible by each of the given numbers.
Co-prime Numbers	Two numbers whose HCF is 1. Example: 8 and 15 are co-prime.

Important Formulas

1. Prime Factorisation Form

Every composite number n can be written as:

$$n = p_1^{a_1} \times p_2^{a_2} \times p_3^{a_3} \times \dots$$

where $p_1 < p_2 < p_3 < \dots$ are primes and $a_1, a_2, a_3, \dots$ are positive integers.

2. HCF by Prime Factorisation

$$\text{HCF} = \text{Product of common prime factors with the lowest powers}$$

3. LCM by Prime Factorisation

$$\text{LCM} = \text{Product of all prime factors with the highest powers}$$

4. Relationship between HCF, LCM and the two numbers

For any two positive integers a and b:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

5. Quick checks

- HCF always divides LCM.
- HCF $\leq$ each of the given numbers.
- LCM $\geq$ each of the given numbers.
- If two numbers are co-prime, their HCF = 1, and LCM = product of the two numbers.

Solved Examples

Example 1 — Basic Prime Factorisation

Q. Find the prime factorisation of 180.

Solution:

Step 1: Divide by the smallest prime. $180 \div 2 = 90$

Step 2: Continue dividing by 2. $90 \div 2 = 45$

Step 3: 45 is not divisible by 2, so try 3. $45 \div 3 = 15$

Step 4: Continue dividing by 3. $15 \div 3 = 5$

Step 5: 5 is itself a prime.

$\therefore\ 180 = 2^2 \times 3^2 \times 5$

Example 2 — Finding HCF and LCM of Two Numbers

Q. Find the HCF and LCM of 72 and 120 using prime factorisation.

Solution:

Step 1: Prime factorise each number. $72 = 2^3 \times 3^2$ $120 = 2^3 \times 3 \times 5$

Step 2: For HCF, take common primes with the **lowest** powers. Common primes: 2 and 3. Lowest powers: 2^3 and 3^1. HCF $= 2^3 \times 3 = 8 \times 3 = 24$

Step 3: For LCM, take all primes with the **highest** powers. All primes: 2, 3, 5. Highest powers: $2^3,\ 3^2,\ 5^1$. LCM $= 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$

Verification: HCF × LCM = 24 × 360 = 8640; also 72 × 120 = 8640 ✓

Example 3 — Using the HCF × LCM = Product Formula

Q. The HCF of two numbers is 12 and their LCM is 360. If one number is 60, find the other.

Solution:

Step 1: Use the relationship formula. $\text{HCF} \times \text{LCM} = a \times b$

Step 2: Substitute known values. $12 \times 360 = 60 \times b$ $4320 = 60b$

Step 3: Solve for b. $b = \frac{4320}{60} = 72$

∴ The other number is **72**.

Example 4 — Word Problem (Application of LCM)

Q. Three bells ring at intervals of 6, 12, and 18 minutes respectively. If they ring together at 9:00 AM, at what time will they next ring together?

Solution:

Step 1: We need the LCM of 6, 12, and 18 — the earliest time all intervals align.

Step 2: Prime factorise each. $6 = 2 \times 3$ $12 = 2^2 \times 3$ $18 = 2 \times 3^2$

Step 3: LCM = product of highest powers of all primes. $\text{LCM} = 2^2 \times 3^2 = 4 \times 9 = 36$

Step 4: They will ring together after 36 minutes.

∴ Next common ringing time = **9:36 AM**.

Example 5 — Three Numbers + HCF (Board-level)

Q. Find the HCF of 96, 144, and 192 using prime factorisation.

Solution:

Step 1: Prime factorise each number. $96 = 2^5 \times 3$ $144 = 2^4 \times 3^2$ $192 = 2^6 \times 3$

Step 2: For HCF, identify common primes across all three. Common primes: 2 and 3.

Step 3: Take the lowest power of each common prime. Lowest power of 2: 2^4 (from 144). Lowest power of 3: 3^1 (from 96 and 192).

$$\text{HCF} = 2^4 \times 3 = 16 \times 3 = 48$$

$$\therefore \text{HCF}(96, 144, 192) = \mathbf{48}$$