Class 10 Maths – Polynomials – Topic: Zeros of Polynomial & Relationship between Coefficients and Zeros (Topicwise Notes)

ALPHA CLASSES DEOBAND | Session 2026–27 | CBSE Board Pattern

Concept – The Big Picture

- A **polynomial** is a neat algebraic expression built from a variable raised to whole-number powers — no negative exponents, no square roots of the variable allowed. Think of it as a well-behaved expression (suljha hua expression).
- The **zero** (shoonya) of a polynomial is the magic value you substitute for the variable that makes the entire expression equal to 0 — it "kills" the polynomial.
- For **quadratic polynomials** $ax^2 + bx + c$, NCERT gives two powerful shortcut formulas that connect the zeroes directly to the coefficients: **sum of zeroes** = $-b/a$ and **product of zeroes** = c/a. No need to solve the equation!
- Using these relations, you can also **form** a quadratic polynomial when only its zeroes are given — just plug sum and product into the formation formula.
- This combined topic is one of the **most scoring** areas in CBSE boards — expect 1-mark MCQs on finding sum/product, 2-mark questions on verifying zeroes, and 3-mark questions on forming polynomials.

Key Definitions

Term	Definition
Polynomial	An algebraic expression of the form $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, where powers of x are non-negative integers and $a_n \neq 0$.
Degree	The highest power of the variable with a non-zero coefficient. Example: degree of $4x^3 - x + 7$ is 3.
Linear Polynomial	A polynomial of degree 1. General form: $ax + b$, where $a \neq 0$. Example: $3x - 5$.
Quadratic Polynomial	A polynomial of degree 2. General form: $ax^2 + bx + c$, where $a \neq 0$. Example: $x^2 - 7x + 10$.
Cubic Polynomial	A polynomial of degree 3. General form: $ax^3 + bx^2 + cx + d$, where $a \neq 0$. Example: $2x^3 + x^2 - 5x + 1$.
Zero of a Polynomial	A value k such that $p(k) = 0$. Geometrically, it is the x-coordinate where the graph crosses the x-axis.
Coefficient	The numerical factor multiplying a particular power of the variable. In $5x^2 - 3x + 2$, the coefficient of x^2 is 5, of x is -3.
Constant Term	The term in a polynomial that has no variable. In $x^2 + 4x - 9$, the constant term is -9.
Sum of Zeroes	For $ax^2 + bx + c$: if zeroes are α and β, then $\alpha + \beta = \frac{-b}{a}$.
Product of Zeroes	For $ax^2 + bx + c$: $\alpha \cdot \beta = \frac{c}{a}$.

Important Formulas

1. Zero of a Polynomial

If $p(x)$ is a polynomial and $p(k) = 0$, then k is called a zero of $p(x)$.

2. Number of Zeroes

A polynomial of degree n can have **at most** n zeroes.

- Linear polynomial → at most 1 zero
- Quadratic polynomial → at most 2 zeroes
- Cubic polynomial → at most 3 zeroes

3. Sum of Zeroes (Quadratic)

For $p(x) = ax^2 + bx + c, a \neq 0$:

$\alpha + \beta = \frac{-b}{a} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

4. Product of Zeroes (Quadratic)

$\alpha \cdot \beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

5. Formation of Quadratic Polynomial from Zeroes

If α and β are the zeroes, then:

$p(x) = x^2 - (\alpha + \beta)\, x + \alpha \cdot \beta$

or equivalently:

$p(x) = x^2 - (\text{sum of zeroes})\, x + (\text{product of zeroes})$

6. Quick Checks

- HCF-style tip: if a polynomial gives $p(k) = 0$, then k is definitely a zero — always verify by substitution when in doubt.
- The graph of a quadratic polynomial is a **parabola**. The zeroes are the **x-intercepts** (where the parabola cuts the x-axis).
- If both zeroes are known, multiply any constant $k \neq 0$ with $x^2 - (\text{sum})x + (\text{product})$ to get infinitely many polynomials with the same zeroes.

Solved Examples

Example 1 — Verifying a Zero by Substitution

Q. Check whether $x = 2$ is a zero of $p(x) = x^2 - 5x + 6$.

Solution:

Step 1: Substitute $x = 2$ into $p(x)$.

$p(2) = (2)^2 - 5(2) + 6$

Step 2: Simplify.

$p(2) = 4 - 10 + 6 = 0$

Step 3: Since $p(2) = 0$, the value $x = 2$ makes the polynomial equal to zero.

$\therefore x = 2$ **is** a zero of $p(x)$. ✓

Example 2 — Finding Sum and Product of Zeroes from Coefficients

Q. Find the sum and product of zeroes of $p(x) = 2x^2 + 7x - 15$.

Solution:

Step 1: Identify the coefficients by comparing with $ax^2 + bx + c$.

$a = 2, \quad b = 7, \quad c = -15$

Step 2: Apply the sum formula.

$\alpha + \beta = \frac{-b}{a} = \frac{-7}{2}$

Step 3: Apply the product formula.

$\alpha \cdot \beta = \frac{c}{a} = \frac{-15}{2}$

∴ Sum of zeroes $= \frac{-7}{2}$, Product of zeroes $= \frac{-15}{2}$.

Example 3 — Forming a Quadratic Polynomial from Given Zeroes

Q. Form a quadratic polynomial whose zeroes are 4 and -3.

Solution:

Step 1: Find the sum of zeroes.

$\alpha + \beta = 4 + (-3) = 1$

Step 2: Find the product of zeroes.

$\alpha \cdot \beta = 4 \times (-3) = -12$

Step 3: Use the formation formula $p(x) = x^2 - (\text{sum})x + (\text{product})$.

$p(x) = x^2 - (1)x + (-12)$

$p(x) = x^2 - x - 12$

Step 4: Quick verification — coefficient check:

Sum $= \frac{-(-1)}{1} = 1$ ✓ and Product $= \frac{-12}{1} = -12$ ✓

∴ The required polynomial is $\boldsymbol{x^2 - x - 12}$.

Example 4 — Non-Monic Polynomial: Verify Coefficient Relations with Known Zeroes

Q. Verify the relationship between the zeroes and coefficients of $p(x) = 3x^2 - 11x - 4$, given that its zeroes are 4 and $\frac{-1}{3}$.

Solution:

Step 1: Identify coefficients.

$a = 3, \quad b = -11, \quad c = -4$

Step 2: Compute sum of zeroes directly.

$\alpha + \beta = 4 + \left(\frac{-1}{3}\right) = \frac{12-1}{3} = \frac{11}{3}$

Step 3: Compute sum using the formula.

$\frac{-b}{a} = \frac{-(-11)}{3} = \frac{11}{3}$

Both match ✓

Step 4: Compute product of zeroes directly.

$\alpha \cdot \beta = 4 \times \left(\frac{-1}{3}\right) = \frac{-4}{3}$

Step 5: Compute product using the formula.

$\frac{c}{a} = \frac{-4}{3}$

Both match ✓

∴ The relationship between zeroes and coefficients is **verified**.

Example 5 — Formation + Complete Verification (Board-Level)

Q. Find a quadratic polynomial whose sum of zeroes is $\frac{3}{2}$ and product of zeroes is $\frac{-1}{2}$. Also find the actual zeroes and verify.

Solution:

Step 1: Use the formation formula.

$p(x) = x^2 - \left(\frac{3}{2}\right)x + \left(\frac{-1}{2}\right)$

$p(x) = x^2 - \frac{3}{2}x - \frac{1}{2}$

Step 2: Multiply throughout by 2 to clear fractions (optional, gives integer coefficients).

$p(x) = 2x^2 - 3x - 1$

Step 3: Find zeroes using the quadratic formula (for verification).

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)} = \frac{3 \pm \sqrt{9+8}}{4} = \frac{3 \pm \sqrt{17}}{4}$$

Step 4: Verify sum of zeroes.

$$\alpha + \beta = \frac{3+\sqrt{17}}{4} + \frac{3-\sqrt{17}}{4} = \frac{6}{4} = \frac{3}{2} \checkmark$$

Step 5: Verify product of zeroes.

$$\alpha \cdot \beta = \frac{(3+\sqrt{17})(3-\sqrt{17})}{16} = \frac{9-17}{16} = \frac{-8}{16} = \frac{-1}{2} \checkmark$$

∴ The required polynomial is $\mathbf{2x^2 - 3x - 1}$ and both relations are verified.

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